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Book Review: *Lattice Gas Hydrodynamics*

Lattice Gas Hydrodynamics. Jean-Pierre Rivet and Jean Pierre Boon, Cambridge University Press, 2001, ISBN - 0 521 41944 1, Cambridge Nonlinear Science Series 11.

Consider a Bravais lattice of vertices with bonds connecting the vertices. Now put point particles on the vertices, with velocities in the directions of the bonds, such that there can be at most one particle with a given velocity at each vertex. Next suppose that at discrete times, each particle moves from one vertex to the next along the direction of its velocity, and when it arrives at the next vertex, its velocity is adjusted according to a set of deterministic or stochastic "collision rules," depending on the number and velocities of the other particles arriving at the same vertex at the same time. After the velocities of all particles are adjusted, the process is repeated. This set of rules, when specified in detail, can be implemented on several kinds of computers using Boolean arithmetic, and the dynamical properties of this system of particles can be described by general methods of statistical mechanics, including methods familiar from the kinetic theory of gases. The systems so described are called lattice gas automata (LGA's), and are the subject of the book under review, *Lattice Gas Hydrodynamics* by J.-P. Rivet and J. P. Boon. In addition to being of considerable interest to workers in non-equilibrium statistical mechanics, cellular automata lattice gases provide very useful, conceptually simple microscopic models for theoretical and computer studies of hydrodynamic flows in fluids. For appropriately constructed models, one expects that the large scale properties of LGA's will be essentially the same as for real fluids, with a possibly significant gain in ease of study.

Lattice gas automata have their origins in the study of discrete velocity models in kinetic theory. A series of very interesting papers by Pomeau, Hardy, and de Pazzis, (HPP), in the 1970's led to work by Frish, Hasslacher, and Pomeau, (FHP), in the 1980's outlining the basic features of the lattice gas automata as we know them today. The FHP work attracted considerable attention in both the scientific and popular literature

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as people realized that these models are well suited for implementation on computers as simple as PC's, and even more so on parallel computers, and they provide a means for simulating hydrodynamic flows past solid objects, hydrodynamic instabilities, and related phenomena.

Lattice Gas Hydrodynamics is devoted to a presentation of the fundamental theoretical ideas and methods used in this field. After the introductory presentation of the lattice gas models and the associated dynamics, the authors turn to the statistical mechanics of such systems. They are careful to contrast the properties of LGA's with those of more realistic fluid models, and they devote considerable attention to a description of those features which result from the discrete nature of the system. As one example of the effects of the discrete nature of the LGA models, the authors note that some simple lattice models do not have the right crystallographic symmetries to provide a correct description of isotropic fluid hydrodynamics, while other models do. They also discuss the various kinds of detailed balance properties that can be imposed on these systems, thus providing an introduction to much of the literature on detailed balance in LGA's and what happens if detailed balance is violated. The chapters on the fundamental theoretical notions develop the subject starting with the lattice Liouville equations and proceeding to the kinetic and hydrodynamic equations needed for the description of the flows. Readers familiar with the kinetic theory of gases will have no trouble recognizing and understanding the methods based upon the traditional Boltzmann equation and more general kinetic equations, albeit with an exclusion principle, required by the condition of single occupation of any particular velocity at each site. Much, but not all of the analysis is based upon the lattice Boltzmann equation, although the Boltzmann approximation is very often invoked at the final stage of calculations to provide explicit results. The theory for hydrodynamic processes in LGA's is developed in some detail using, among others, multiple time scale methods, and clearly motivated, so that readers unfamiliar with these methods can follow and understand them in this as well as in other settings. It would have been nice to have a chapter on BBGKY-hierarchy methods as applied to LGA's since they provide some insights into mode coupling theories. For that one will have to refer to the literature on the subject, much of which is mentioned in the final chapter of the book.

After treating the theoretical foundations, the authors develop the main themes of the book—a description of the average hydrodynamic properties of the LGA fluids and of the fluctuations taking place in them, including equilibrium thermal fluctuations of the Landau–Placzek type. The Green–Kubo relations which play an important role in the theory and computer simulations for molecular fluids are discussed in this context as well, with some pages describing time-correlation functions for LGA systems. One striking result obtained using LGA's is the computer verification of the presence of long time tails in equilibrium time-correlation functions, a topic of some interest to workers in non-equilibrium statistical mechanics, since these time-correlation functions are related to transport coefficients through the Green–Kubo formulae.

The book concludes with two descriptive chapters. The results of computer simulations of hydrodynamic flows in LGA fluids are described in Chapter 10, along with some details about special purpose and other computers used to simulate LGA's, as well as methods for constructing efficient algorithms. The book concludes with a guide to the literature on LGA fluids, with a listing of key papers on about twenty subtopics, such as fluid mixtures, reaction diffusion systems, systems violating detailed balance, as well as a listing of important review articles and of four other recent books on this topic. The latter include recent books by D. Rothman and S. Zaleski (1997), by B. Chopard and M. Droz (1999), by D. Wolf-Gladrow (2000), and by S. Succi (2001), all of which treat the basic statistical mechanics of cellular automata lattice gases, but with emphases on somewhat different topics, largely dictated by the research interests of the authors, as might be expected.

As a person who has done only a small amount of work in this area, and none of it on the central issues, I enjoyed reading, and learned a lot from this book. I suggest that readers new to the subject read Chapters 2 and 3 together, since the import of some of the discussion in Chapter 2 becomes clearer when one sees how symmetries, detailed balance assumptions, etc., discussed in Chapter 2, affect the dynamics on particular lattices or for specific collision models, described in Chapter 3. In any case, this book is certainly an excellent introduction to the physics of lattice gas automata and to the hydrodynamic flows that take place in them.

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